

Constraining the density slope of nuclear symmetry energy at subsaturation densities using electric dipole polarizability in ^{208}Pb

Zhen Zhang¹ and Lie-Wen Chen^{*1,2}

¹*Department of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China*

²*Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*
(Dated: December 12, 2014)

Nuclear structure observables usually most effectively probe the properties of nuclear matter at subsaturation densities rather than at saturation density. We demonstrate that the electric dipole polarizability α_D in ^{208}Pb is sensitive to both the magnitude $E_{\text{sym}}(\rho_c)$ and density slope $L(\rho_c)$ of the symmetry energy at the subsaturation cross density $\rho_c = 0.11 \text{ fm}^{-3}$. Using the experimental data of α_D in ^{208}Pb from RCNP and the recent accurate constraint of $E_{\text{sym}}(\rho_c)$ from the binding energy difference of heavy isotope pairs, we extract a value of $L(\rho_c) = 47.3 \pm 7.8 \text{ MeV}$. The implication of the present constraint of $L(\rho_c)$ to the symmetry energy at saturation density, the neutron skin thickness of ^{208}Pb and the core-crust transition density in neutron stars is discussed.

PACS numbers: 21.65.Ef, 24.30.Cz, 21.60.Jz, 21.30.Fe

I. INTRODUCTION

Due to its multifaceted roles in nuclear physics and astrophysics [1–4] as well as new physics beyond the standard model [5–8], the symmetry energy has become a hot topic in current research frontiers of nuclear physics and astrophysics [9]. During the last decade, a lot of experimental, observational and theoretical efforts have been devoted to constraining the magnitude $E_{\text{sym}}(\rho)$ and density slope $L(\rho)$ of the symmetry energy at nuclear saturation density ρ_0 ($\sim 0.16 \text{ fm}^{-3}$), i.e., $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$. Although important progress has been made, large uncertainties on the values of $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ still exist (See, e.g., Refs. [2–4, 9–14]). For instance, while the $E_{\text{sym}}(\rho_0)$ is determined to be around $32 \pm 4 \text{ MeV}$, the extracted $L(\rho_0)$ varies significantly from about 20 to 115 MeV, depending on the observables and analysis methods. To better understand the model dependence and narrow the uncertainties of the constraints is thus of extreme importance.

While many studies on heavy ion collisions and neutron stars have significantly improved our knowledge on the symmetry energy, more and more constraints on the symmetry energy have been obtained in recent years from analyzing the properties of finite nuclei, such as the nuclear binding energy [15–19], the neutron skin thickness [20–22], and the resonances and excitations [23–31]. Furthermore, it has been realized that the properties of finite nuclei usually provide more precise constraints on $E_{\text{sym}}(\rho)$ and $L(\rho)$ at subsaturation densities rather than at saturation density ρ_0 . This feature is understandable since the characteristic (average) density of finite nuclei is less than ρ_0 . For example, the average density of heavy nuclei (e.g., ^{208}Pb) is about 0.11 fm^{-3} , and thus the prop-

erties of heavy nuclei most effectively probe the properties of nuclear matter around 0.11 fm^{-3} [32–42]. Indeed, a quite accurate constraint on the symmetry energy at the subsaturation cross density $\rho_c = 0.11 \text{ fm}^{-3}$, i.e., $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20 \text{ MeV}$, has been recently obtained from analyzing the binding energy difference of heavy isotope pairs [36]. In contrast to the fact that many and precise constraints on the magnitude of $E_{\text{sym}}(\rho)$ around ρ_c have been obtained, to the best of our knowledge, so far there is only one experimental constraint on the density slope $L(\rho_c)$ which was obtained from analyzing the neutron skin data of Sn isotopes [36]. Knowledge on $L(\rho_c)$ is not only important for understanding the density dependence of the symmetry energy itself, but also plays a central role in determining the neutron skin thickness of heavy nuclei and the core-crust transition density in neutron stars. Therefore, any new constraints on $L(\rho_c)$ will be extremely useful.

In the present work, with the precise knowledge of $E_{\text{sym}}(\rho_c)$, we demonstrate that the electric dipole polarizability α_D in ^{208}Pb measured at the Research Center for Nuclear Physics (RCNP) via polarized proton inelastic scattering at forward angles, can put a strong limit on the $L(\rho_c)$. We emphasize since at forward angles Coulomb excitation dominates, the extracted α_D at RCNP is expected to be a relatively clean isovector indicator with less uncertainties from strong interaction.

II. MODEL AND METHOD

A. The symmetry energy and Skyrme-Hartree-Fock approach

The equation of state (EOS) of asymmetric nuclear matter, given by its binding energy per nucleon, can be written as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad (1)$$

*Corresponding author (email: lwchen@sjtu.edu.cn)

where ρ is the baryon density, $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is isospin asymmetry, $E_0(\rho) = E(\rho, \delta = 0)$ is the EOS of symmetric nuclear matter, and the symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0}. \quad (2)$$

Around a reference density ρ_r , the $E_{\text{sym}}(\rho)$ can be expanded in $\chi_r = (\rho - \rho_r)/\rho_r$ as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_r) + \frac{L(\rho_r)}{3} \chi_r + O(\chi_r^2), \quad (3)$$

where $L(\rho_r) = 3\rho_r \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \Big|_{\rho=\rho_r}$ is the density slope parameter which characterizes the density dependence of the symmetry energy around ρ_r .

Our calculations in the present work are based on the Skyrme-Hartree-Fock (SHF) approach with the so-called standard Skyrme force (see, e.g., Ref. [43–45]) which includes 10 parameters, i.e., the 9 Skyrme force parameters σ , $t_0 - t_3$, $x_0 - x_3$, and the spin-orbit coupling constant W_0 . Instead of directly using the 9 Skyrme force parameters, we can express them explicitly in terms of 9 macroscopic quantities, i.e., ρ_0 , $E_0(\rho_0)$, the incompressibility K_0 , the isoscalar effective mass $m_{s,0}^*$, the isovector effective mass $m_{v,0}^*$, $E_{\text{sym}}(\rho_r)$, $L(\rho_r)$, the gradient coefficient G_S , and the symmetry-gradient coefficient G_V . In this case, we can examine the correlation of properties of finite nuclei with each individual macroscopic quantity by varying individually these macroscopic quantities within their empirical ranges. Recently, this correlation analysis method has been successfully applied to study nuclear matter properties from analyzing nuclear structure observables [21, 33, 36, 46, 47], and will also be used in this work.

B. Random-phase approximation and electric dipole polarizability

The random-phase approximation (RPA) provides an important microscopic approach to calculate the electric dipole polarizability in finite nuclei. Within the framework of RPA theory, for a given excitation operator \hat{F}_{JM} , the reduced transition probability from RPA ground state $|\tilde{0}\rangle$ to RPA excitation state $|\nu\rangle$ is given by:

$$B(EJ : \tilde{0} \rightarrow |\nu\rangle) = |\langle \nu | \hat{F}_J | \tilde{0} \rangle|^2 = \left| \sum_{mi} (X_{mi}^\nu + Y_{mi}^\nu) \langle m | \hat{F}_J | i \rangle \right|^2, \quad (4)$$

where $m(i)$ denotes the unoccupied (occupied) single nucleon state; $\langle m | \hat{F}_J | i \rangle$ is the reduced matrix element of \hat{F}_{JM} ; and X_{mi}^ν and Y_{mi}^ν are the RPA amplitudes. The strength function then can be calculated as:

$$S(E) = \sum_{\nu} |\langle \nu | \hat{F}_J | \tilde{0} \rangle|^2 \delta(E - E_\nu), \quad (5)$$

where E_ν is the energy of RPA excitation state $|\nu\rangle$. Thus the moments of strength function can be obtained as:

$$m_k = \int dE E^k S(E) = \sum_{\nu} |\langle \nu | \hat{F}_J | \tilde{0} \rangle|^2 E_\nu^k. \quad (6)$$

In the case of electric dipole (E1) response, the excitation operator is defined as:

$$\hat{F}_{1M} = \frac{eN}{A} \sum_{i=1}^Z r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A} \sum_{i=1}^N r_i Y_{1M}(\hat{r}_i), \quad (7)$$

where Z , N and A are proton, neutron and mass number, respectively; r_i is the nucleon's radial coordinate; $Y_{1M}(\hat{r}_i)$ is the corresponding spherical harmonic function. For a given Skyrme interaction, we can calculate the inverse energy-weighted moment m_{-1} using the HF-RPA method, and then obtain the electric dipole polarizability α_D as

$$\alpha_D = \frac{8\pi}{9} e^2 \int dE E^{-1} S(E) = \frac{8\pi}{9} e^2 m_{-1}. \quad (8)$$

For the theoretical calculations of electric dipole polarizability in ^{208}Pb in the present work, we employ the Skyrme-RPA program by Colò *et al* [48]. In this program, the SHF mean field and the RPA excitations are fully self-consistent. In particular, we calculate the isovector dipole strength in ^{208}Pb with a spherical box extending up to 24 fm, a radial mesh of 0.1 fm and a cutoff energy of $E_C = 150$ MeV which denotes the maximum energy of the unoccupied single-particle states in the RPA model space. Then the inverse energy-weighted moment m_{-1} is evaluated with an upper integration limit of 130 MeV according to the experimental energy range [30], and thus the electric dipole polarizability α_D can be calculated invoking Eq. (8).

C. The symmetry energy and electric dipole polarizability

The electric dipole polarizability α_D has been shown to be a sensitive probe of the symmetry energy [26, 27, 31]. In particular, based on the droplet model, Roca-Maza *et al.* [31] obtained the following relation:

$$\alpha_D = \frac{\pi e^2}{54} \frac{A \langle r^2 \rangle}{E_{\text{sym}}(\rho_0)} \left[1 + \frac{5}{3} \frac{E_{\text{sym}}(\rho_0) - a_{\text{sym}}(A)}{E_{\text{sym}}(\rho_0)} \right], \quad (9)$$

where $\langle r^2 \rangle$ is the mean-square radius and $a_{\text{sym}}(A)$ is the symmetry energy coefficient of a finite nucleus of mass number A . Furthermore, using the empirical relation $a_{\text{sym}}(A) \approx E_{\text{sym}}(\rho_A)$ [32, 33, 37] and expanding $E_{\text{sym}}(\rho_A)$ as

$$E_{\text{sym}}(\rho_A) \approx E_{\text{sym}}(\rho_0) - L(\rho_0)(\rho_0 - \rho_A)/3\rho_0, \quad (10)$$

Roca Maza *et al.* demonstrated that α_D is correlated with both $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$. Particularly, based on a

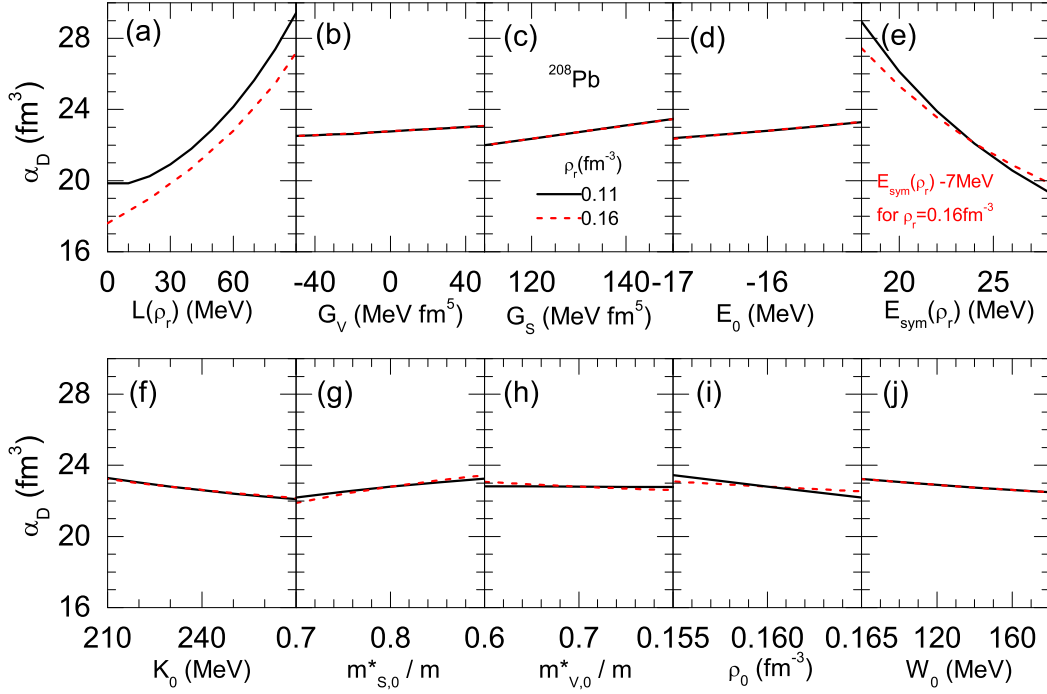


FIG. 1: (Color online) The electric dipole polarizability α_D in ^{208}Pb from SHF-RPA calculations with the MSL0 interaction by varying individually $L(\rho_r)$ (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\text{sym}}(\rho_r)$ (e), K_0 (f), $m_{s,0}^*$ (g), $m_{v,0}^*$ (h), ρ_0 (i), and W_0 (j) for $\rho_r = 0.11$ and 0.16 fm^{-3} . The $E_{\text{sym}}(\rho_r)$ is shifted by subtracting 7 MeV for $\rho_r = 0.16 \text{ fm}^{-3}$.

large and representative set of relativistic and nonrelativistic nuclear mean-field models, they found a strong linear correlation between $\alpha_D E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ and then extracted the constraint $L(\rho_0) = 43 \pm (6)_{\text{expt}} \pm (8)_{\text{theor}} \pm (12)_{\text{est}}$ MeV from the combination of the experimental determination of α_D with the empirical estimate of $E_{\text{sym}}(\rho_0) = 31 \pm (2)_{\text{est}}$ MeV. One can see that the uncertainty of the estimated $E_{\text{sym}}(\rho_0)$ leads to a large error of 12 MeV for $L(\rho_0)$.

Instead of expressing $E_{\text{sym}}(\rho_A)$ in terms of $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ as in Eq. (10), one can also express $E_{\text{sym}}(\rho_0)$ in terms of $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ as

$$E_{\text{sym}}(\rho_0) \approx E_{\text{sym}}(\rho_c) + L(\rho_c)(\rho_0 - \rho_c)/3\rho_c. \quad (11)$$

Noting $\rho_{208} \approx \rho_c$ [32, 33, 37], one can then see from Eqs. (9) and (11) that α_D in ^{208}Pb is also correlated with both $L(\rho_c)$ and $E_{\text{sym}}(\rho_c)$. As we will see in the following, the microscopic RPA calculations indeed show that α_D is sensitive to $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ as well as to $L(\rho_c)$ and $E_{\text{sym}}(\rho_c)$. Since $E_{\text{sym}}(\rho_c)$ has been stringently constrained recently (see, e.g., $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV in Ref.[36]), the α_D in ^{208}Pb can thus be used to constrain the $L(\rho_c)$ parameter.

III. RESULTS AND DISCUSSIONS

To examine the correlation of the α_D in ^{208}Pb with each macroscopic quantity, especially on $E_{\text{sym}}(\rho_r)$ and

$L(\rho_r)$, we show in Fig. 1 the α_D in ^{208}Pb from SHF with the Skyrme force MSL0 [21] by varying individually $L(\rho_r)$, G_V , G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_r)$, K_0 , $m_{s,0}^*$, $m_{v,0}^*$, ρ_0 , and W_0 within their empirical uncertain ranges, namely, varying one quantity at a time while keeping all others at their default values in MSL0, for $\rho_r = 0.11$ and 0.16 fm^{-3} , respectively. It is seen from Fig. 1 that, as Eq. (9) suggests, the α_D in ^{208}Pb exhibits strong correlations with both $L(\rho_r)$ and $E_{\text{sym}}(\rho_r)$, while much weaker correlation with other macroscopic quantities. Particularly, the α_D decreases sensitively with $E_{\text{sym}}(\rho_r)$ while increases rapidly with $L(\rho_r)$, implying a fixed value of α_D will lead to a strong positive correlation between $E_{\text{sym}}(\rho_r)$ and $L(\rho_r)$. The results for $\rho_r = 0.16 \text{ fm}^{-3}$ just confirm the correlations of α_D with $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ reported in Ref.[31]. For $\rho_r = 0.11 \text{ fm}^{-3}$, given that the symmetry energy at $\rho_c = 0.11 \text{ fm}^{-3}$ has been well constrained as $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV, one thus expects the α_D in ^{208}Pb can constrain stringently the parameter $L(\rho_c)$.

Fixing the values of other 8 macroscopic quantities, i.e., G_V , G_S , $E_0(\rho_0)$, K_0 , $m_{s,0}^*$, $m_{v,0}^*$, ρ_0 and W_0 at their default values in MSL0, we illustrate in Fig. 2 by open up-triangles (down-triangles) the α_D in ^{208}Pb as a function of $L(\rho_c)$ for $E_{\text{sym}}(\rho_c) = 26.45$ (26.85) MeV. As expected, it is seen from Fig. 2 that the α_D in ^{208}Pb increases (decreases) with $L(\rho_c)$ ($E_{\text{sym}}(\rho_c)$) for a fixed $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$). By comparing with the experimental data $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$, one can extract a strong con-

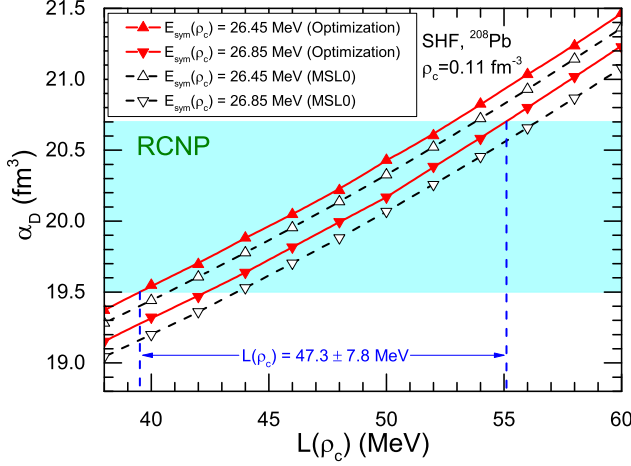


FIG. 2: (Color online) The electric dipole polarizability α_D in ^{208}Pb as a function of $L(\rho_c)$ for fixed $E_{\text{sym}}(\rho_c)$. The open (solid) up- and down-triangles represent the results with $E_{\text{sym}}(\rho_c) = 26.45$ and 26.85 MeV, respectively, from SHF-RPA calculations with the values of other parameters fixed in MSL0 (obtained in optimization). The band indicates the experimental value of $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$ from RCNP [30].

straint of $L(\rho_c) = 48.6 \pm 7.9 \text{ MeV}$.

The above constraint of $L(\rho_c) = 48.6 \pm 7.9 \text{ MeV}$ has been obtained by neglecting the weak correlations between the α_D in ^{208}Pb and other 8 macroscopic quantities. To test the robustness of this constraint and to obtain a more precise constraint, for fixed $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$, we optimize all other 8 parameters instead of simply fixing them at their default values in MSL0, by minimizing the weighted sum of χ^2 evaluated from the difference between SHF prediction and the experimental data for some selected observables using the simulated annealing technique [49]. In particular, in the optimization, we chose the following experimental data of spherical even-even nuclei, i.e., (i) the binding energy E_B of ^{16}O , $^{40,48}\text{Ca}$, $^{56,68}\text{Ni}$, ^{88}Sr , ^{90}Zr , $^{100,116,132}\text{Sn}$, ^{144}Sm , ^{208}Pb [50]; (ii) the charge rms radii r_C of ^{16}O , $^{40,48}\text{Ca}$, ^{56}Ni , ^{88}Sr , ^{90}Zr , $^{116,132}\text{Sn}$, ^{144}Sm , ^{208}Pb [51, 52]; (iii) the breathing mode energy E_0 of ^{90}Zr , ^{116}Sn , ^{144}Sm and ^{208}Pb [53]. In the calculation of the breathing mode energy $E_0 = \sqrt{m_1/m_{-1}}$, we evaluate the inverse energy-weighted sum rule m_{-1} with the constrained Hartree-Fock (CHF) method and obtain the energy-weighted sum rule m_1 using the double commutator sum rule [54–57]. In addition, in the optimization, we constrain the macroscopic parameters by requiring that (i) the neutron $3p_{1/2} - 3p_{3/2}$ energy level splitting in ^{208}Pb should lie in the range of $0.8 - 1.0 \text{ MeV}$; (ii) $m_{s,0}^*$ should be greater than $m_{v,0}^*$ and here we set $m_{s,0}^* - m_{v,0}^* = 0.1m$ (m is nucleon mass in vacuum) to be consistent with the extraction from global nucleon optical potentials constrained by world data on nucleon-nucleus and (p,n) charge-exchange reactions [58]. As usual, in the optimization, we assign a theoretical error 1.2 MeV to E_B , 0.025 fm to r_C while use the experimental error

for breathing mode energy E_0 with a weight factor 0.08 , so that the respective χ^2 evaluated from each sort of experimental data is roughly equal to the number of the corresponding data points [59].

Using the above optimization process, we evaluate the electric dipole polarizability α_D in ^{208}Pb as a function of $L(\rho_c)$ for a fixed $E_{\text{sym}}(\rho_c)$, and the results are shown in Fig 2 by solid up-triangles (down-triangles) for $E_{\text{sym}}(\rho_c) = 26.45$ (26.85) MeV. It should be noted that for each pair of $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ with fixed values, the other 8 macroscopic quantities have been optimized accordingly as described above. It is interesting to see that the values of α_D with optimization are quite consistent with the results using the default values in MSL0 without optimization and only show a small upward shift compared with the latter. Comparing the results from optimization to the experimental data, one can obtain a constraint of $L(\rho_c) = 47.3 \pm 7.8 \text{ MeV}$, which is again in good agreement with the constraint $L(\rho_c) = 48.6 \pm 7.9 \text{ MeV}$ extracted using the default values in MSL0. These features demonstrate the validity of neglecting the weak correlations between the α_D in ^{208}Pb and other 8 macroscopic quantities. The present constraint on $L(\rho_c)$ further agrees very well with the constraint $L(\rho_c) = 46.0 \pm 4.5 \text{ MeV}$ extracted from analyzing the experimental data on the neutron skin thickness of Sn isotopes [36]. This is a very interesting finding since these two constraints are obtained from two completely independent experimental observables.

In addition, using the constrained $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ together with the corresponding 8 other optimized quantities, one can easily extract the $E_{\text{sym}}(\rho)$ and $L(\rho)$ at saturation density ρ_0 , and the results are $E_{\text{sym}}(\rho_0) = 32.7 \pm 1.7 \text{ MeV}$ and $L(\rho_0) = 47.1 \pm 17.7 \text{ MeV}$, which are essentially consistent with other constraints extracted from terrestrial experiments, astrophysical observations, and theoretical calculations with controlled uncertainties [10–13, 60]. Especially, our present results agree surprisingly well with the constraint of $E_{\text{sym}}(\rho_0) = 31.2 - 34.3 \text{ MeV}$ and $L(\rho_0) = 36 - 55 \text{ MeV}$ (at 95% confidence level) obtained from analyzing the mass and radius of neutron stars [61] as well as that of $E_{\text{sym}}(\rho_0) = 29.0 - 32.7 \text{ MeV}$ and $L(\rho_0) = 40.5 - 61.9 \text{ MeV}$ extracted from the experimental, theoretical and observational analyses [11]. Our results are also in agreement with the constraint of $E_{\text{sym}}(\rho_0) = 32.0 \pm 1.8 \text{ MeV}$ and $L(\rho_0) = 43.1 \pm 15 \text{ MeV}$ from analyzing pygmy dipole resonances (PDR) of $^{130,132}\text{Sn}$ [24] and that of $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.3 \text{ MeV}$ and $L(\rho_0) = 64.8 \pm 15.7 \text{ MeV}$ from analyzing PDR of ^{68}Ni and ^{132}Sn [25]. In addition, our results are further consistent with the constraint of $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.0 \text{ MeV}$ and $L(\rho_0) = 45.2 \pm 10.0 \text{ MeV}$ extracted from analyzing the experimental data of the binding energy difference of heavy isotope pairs and the neutron skins of Sn isotopes [36] as well as the constraint of $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5 \text{ MeV}$ and $L(\rho_0) = 70 \pm 15 \text{ MeV}$ from a new finite-range droplet model analysis of the nuclear mass [18].

Given that the neutron skin thickness Δr_{np} of ^{208}Pb is

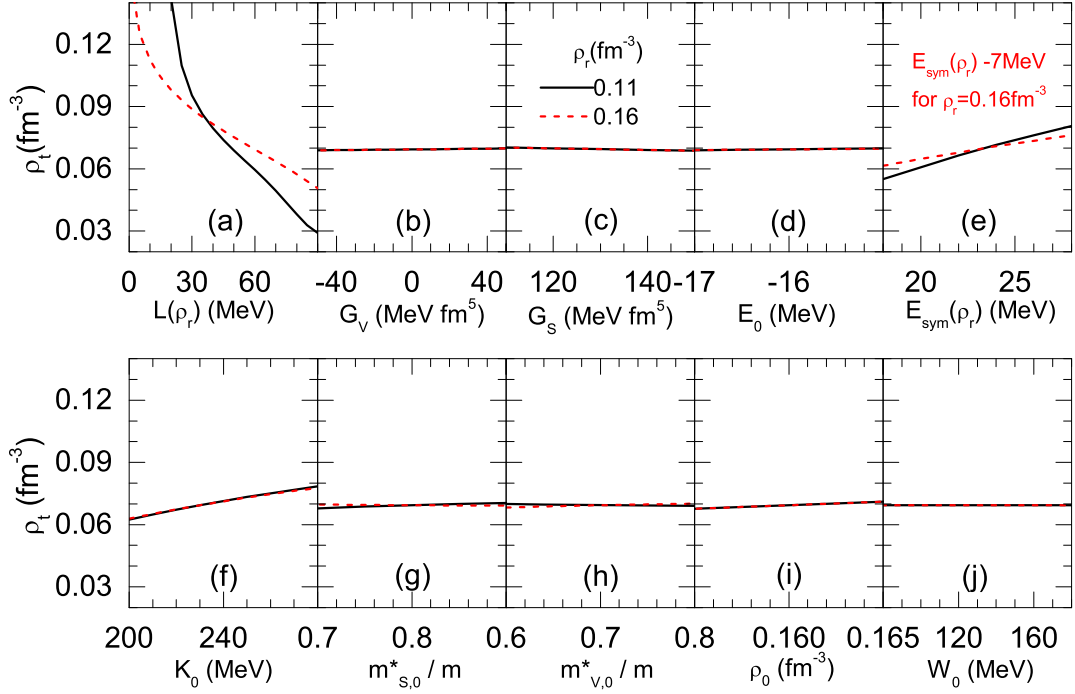


FIG. 3: (Color online) Same as Fig. 1 but for the core-crust transition density ρ_t in neutron stars.

uniquely fixed by the slope parameter $L(\rho_c)$ at $\rho_c = 0.11 \text{ fm}^{-3}$ [36], we can also extract a constraint $\Delta r_{np} = 0.176 \pm 0.027 \text{ fm}$ for ^{208}Pb by using the optimized parameters together with $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20 \text{ MeV}$ and $L(\rho_c) = 47.3 \pm 7.8 \text{ MeV}$. Our result is consistent with the estimated range $\Delta r_{np} = 0.165 \pm (0.009)_{\text{expt}} \pm (0.013)_{\text{theor}} \pm (0.021)_{\text{est}} \text{ fm}$ in Ref. [31] obtained by analyzing the experimental α_D in ^{208}Pb with an empirical range of $E_{\text{sym}}(\rho_0) = 31 \pm (2)_{\text{est}}$. One can see that our present constraint on Δr_{np} of ^{208}Pb has higher precision, indicating a more precise constraint on the symmetry energy at a subsaturation density is very helpful to extract Δr_{np} of ^{208}Pb from the electric dipole polarizability. Our result further agrees with the constraint $\Delta r_{np} = 0.156^{+0.025}_{-0.021} \text{ fm}$ obtained from the ^{208}Pb dipole polarizability by using an empirical correlation between α_D and Δr_{np} of ^{208}Pb [30], the constraint $\Delta r_{np} = 0.15 \pm 0.03(\text{stat.})^{+0.01}_{-0.03}(\text{sys.}) \text{ fm}$ extracted very recently from coherent pion photoproduction cross sections [62], and within the experimental error bar the constraint $\Delta r_{np} = 0.33^{+0.16}_{-0.18} \text{ fm}$ extracted from the PREX at JLab [63].

Furthermore, it has been well established that the core-crust transition density ρ_t in neutron stars, which plays a crucial role in neutron star properties [1], is strongly correlated with the density slope $L(\rho_0)$ of the symmetry energy (see, e.g., Ref. [64]). In particular, in Ref. [21], the same correlation analysis method as in this work has been successfully applied to study the correlation between ρ_t and the various macroscopic quantities, and indeed a strong correlation between ρ_t and $L(\rho_0)$ has been

found. As mentioned in Ref. [36], a similar strong correlation is also existed between ρ_t and $L(\rho_c)$, and this is demonstrated in Fig. 3 which shows the same correlations as Fig. 1 but for the core-crust transition density ρ_t in neutron stars. Here, the transition density ρ_t is calculated by using a dynamical approach (see, e.g., Ref. [64]). One can see from Fig. 3 that, for both $\rho_r = 0.11$ and 0.16 fm^{-3} , ρ_t exhibits a strong correlation with $L(\rho_r)$, a weak dependence on $E_{\text{sym}}(\rho_r)$ and K_0 , but almost no sensitivity to other macroscopic parameters. Employing the optimized values for other macroscopic parameters as well as $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20 \text{ MeV}$ and $L(\rho_c) = 47.3 \pm 7.8 \text{ MeV}$, we then obtain a value of $\rho_t = 0.084 \pm 0.009 \text{ fm}^{-3}$, which agrees well with the empirical values [1].

IV. SUMMARY AND OUTLOOK

In summary, we have demonstrated that the electric dipole polarizability α_D in ^{208}Pb is sensitive to both the magnitude $E_{\text{sym}}(\rho_c)$ and density slope $L(\rho_c)$ of the symmetry energy at a subsaturation cross density $\rho_c = 0.11 \text{ fm}^{-3}$, and it decreases (increases) with $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$), leading to a positive correlation between $L(\rho_c)$ and $E_{\text{sym}}(\rho_c)$ for a fixed value of α_D in ^{208}Pb . Using the experimental value of α_D in ^{208}Pb measured at RCNP and the very well-constrained range of $E_{\text{sym}}(\rho_c)$, we have obtained a strong constraint on the slope parameter $L(\rho_c) = 47.3 \pm 7.8 \text{ MeV}$. This constraint is in surprisingly good agreement with the previous solely existing constraint $L(\rho_c) = 46.0 \pm 4.5 \text{ MeV}$ from neutron

skin data of Sn isotopes, demonstrating the robustness of these constraints on the value of the $L(\rho_c)$ parameter.

The present constraint of $L(\rho_c)$ further leads to $E_{\text{sym}}(\rho_0) = 32.7 \pm 1.7$ MeV and $L(\rho_0) = 47.1 \pm 17.7$ MeV for the symmetry energy at saturation density, the neutron skin thickness $\Delta r_{\text{np}} = 0.176 \pm 0.027$ fm for ^{208}Pb , and $\rho_t = 0.084 \pm 0.009$ fm $^{-3}$ for the core-crust transition density of neutron stars. These results are nicely consistent with many other constraints extracted from terrestrial experiments, astrophysical observations, and theoretical calculations with controlled uncertainties.

Our present results are based on the standard SHF energy density functional. It will be interesting to see how the results change if different energy-density functionals, e.g., the relativistic mean field model or the extended non-standard SHF energy density functional, are applied. These works are in progress and will be reported elsewhere.

Acknowledgments

We are grateful to Li-Gang Cao for helpful discussions on the Skyrme-RPA code. This work was supported in part by the Major State Basic Research Development Program (973 Program) in China under Contract Nos. 2015CB856904 and 2013CB834405, the NNSF of China under Grant Nos. 11135011 and 11275125, the “Shu Guang” project supported by Shanghai Municipal Education Commission and Shanghai Education Development Foundation, the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning, and the Science and Technology Commission of Shanghai Municipality (11DZ2260700).

-
- [1] J.M. Lattimer and M. Prakash, *Science* **304**, 536 (2004); *Phys. Rep.* **442**, 109 (2007).
 - [2] A.W. Steiner, M. Prakash, J.M. Lattimer, and P.J. Ellis, *Phys. Rep.* **411**, 325 (2005).
 - [3] V. Baran, M. Colonna, V. Greco, and M. Di Toro, *Phys. Rep.* **410**, 335 (2005).
 - [4] B.A. Li, L.W. Chen, and C.M. Ko, *Phys. Rep.* **464**, 113 (2008).
 - [5] C.J. Horowitz, S.J. Pollock, P.A. Souder, and R. Michaels, *Phys. Rev. C* **63**, 025501 (2001).
 - [6] T. Sil, M. Centelles, X. Viñas, and J. Piekarewicz, *Phys. Rev. C* **71**, 045502 (2005).
 - [7] D.H. Wen, B.A. Li, and L.W. Chen, *Phys. Rev. Lett.* **103**, 211102 (2009).
 - [8] H. Zheng, Z. Zhang, and L.W. Chen, *J. Cosmo. Astropart. Phys.* **08**, 011 (2014) [arXiv:1403.5134].
 - [9] Topical Issue Nuclear Symmetry Energy edited by B.A. Li, A. Ramos, G. Verde, I. Vidana, *Eur. Phys. J. A* **50**, (2014).
 - [10] B.M. Tsang *et al.*, *Phys. Rev. C* **86**, 015803 (2012).
 - [11] J.M. Lattimer, *Ann. Rev. Nucl. Part. Sci.* **62**, 485 (2012).
 - [12] L.W. Chen, Nuclear Structure in China 2012: Proceedings of the 14th National Conference on Nuclear Structure in China (NSC2012) (World Scientific, Singapore, 2012), pp. 43-54 [arXiv:1212.0284].
 - [13] B.A. Li *et al.*, *J. Phys.: Conf. Series* **413**, 012021 (2013) [arXiv:1212.1178].
 - [14] C.J. Horowitz *et al.*, *J. Phys. G* **41**, 093001 (2014).
 - [15] W.D. Myers and W.J. Swiatecki, *Nucl. Phys.* **A601**, 141 (1996).
 - [16] P. Danielewicz and J. Lee, *Nucl. Phys.* **A818**, 36 (2009).
 - [17] M. Liu, N. Wang, Z. X. Li, and F. S. Zhang, *Phys. Rev. C* **82**, 064306 (2010).
 - [18] P. Möller, W. D. Myers, H. Sagawa, and S. Yoshida, *Phys. Rev. Lett.* **108**, 052501 (2012).
 - [19] J.M. Lattimer and Y. Lim, *Astrophys. J.* **771**, 51 (2013).
 - [20] M. Warda, X. Vinas, X. Roca-Maza, and M. Centelles, *Phys. Rev. C* **80**, 024316 (2009).
 - [21] L.W. Chen, C.M. Ko, B.A. Li, and J. Xu, *Phys. Rev. C* **82**, 024321 (2010).
 - [22] X. Vinas, M. Centelles, X. Roca-Maza, and M. Warda, *Eur. Phys. J. A* **50**, 27 (2014).
 - [23] H. Sagawa, S. Yoshida, X.-R. Zhou, K. Yako, and H. Sakai, *Phys. Rev. C* **76**, 024301 (2007).
 - [24] A. Klimkiewicz *et al.*, *Phys. Rev. C* **76**, 051603 (2007).
 - [25] A. Carbone *et al.*, *Phys. Rev. C* **81**, 041301(R) (2010).
 - [26] P.-G. Reinhard and W. Nazarewicz, *Phys. Rev. C* **81**, 051303(R) (2010).
 - [27] J. Piekarewicz *et al.*, *Phys. Rev. C* **85**, 041302(R) (2012).
 - [28] J. Piekarewicz, *Eur. Phys. J. A* **50**, 25 (2014).
 - [29] G. Colò, U. Garg, and H. Sagawa, *Eur. Phys. J. A* **50**, 26 (2014).
 - [30] A. Tamii *et al.*, *Phys. Rev. Lett.* **107**, 062502 (2011).
 - [31] X. Roca-Maza *et al.*, *Phys. Rev. C* **88**, 024316 (2013).
 - [32] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, *Phys. Rev. Lett.* **102**, 122502 (2009).
 - [33] L.W. Chen, *Phys. Rev. C* **83**, 044308 (2011).
 - [34] E. Khan, J. Margueron, and I. Vidana, *Phys. Rev. Lett.* **109**, 092501 (2012).
 - [35] F.J. Fattoyev, W.G. Newton, and B.A. Li, *Phys. Rev. C* **90**, 022801(R) (2014).
 - [36] Z. Zhang and L.W. Chen, *Phys. Lett.* **B726**, 234 (2013).
 - [37] P. Danielewicz and J. Lee, *Nucl. Phys.* **A922**, 1 (2014).
 - [38] L. Trippa, G. Colò, and E. Vigezzi, *Phys. Rev. C* **77**, 061304(R) (2008).
 - [39] L.G. Cao and Z.Y. Ma, *Chin. Phys. Lett.* **25**, 1625 (2008).
 - [40] X. Roca-Maza *et al.*, *Phys. Rev. C* **87**, 034301 (2013).
 - [41] B.A. Brown, *Phys. Rev. Lett.* **11**, 232502 (2013).
 - [42] N. Wang, L. Ou, and M. Liu, *Phys. Rev. C* **87**, 034327 (2013).
 - [43] E. Chabanat *et al.*, *Nucl. Phys.* **A627**, 710 (1997).
 - [44] J. Friedrich and P.-G. Reinhard, *Phys. Rev. C* **33**, 335 (1986).
 - [45] P. Klüpfel *et al.*, *Phys. Rev. C* **79**, 034310 (2009).
 - [46] L.W. Chen and J.Z. Gu, *J. Phys. G* **39**, 035104 (2012).
 - [47] L.W. Chen, *Sci. China: Phys. Mech. Astro.* **54** (Suppl. 1), s124 (2011).
 - [48] G. Colò, L.G. Cao, N. Van Giai, and L. Capelli, *Comput. Phys. Commun.* **184**, 142 (2013).
 - [49] B.K. Agrawal, S. Shlomo, and V. Kim Au, *Phys. Rev. C*

- 72**, 014310 (2005).
- [50] M. Wang *et al.*, Chin. Phys. C **36** (2012) 1287.
 - [51] I. Angeli, At. Data Nucl. Data. Tab. **87** (2004) 185.
 - [52] F. Le Blanc *et al.*, Phys. Rev. C **72**, (2005) 034305.
 - [53] D.H. Youngblood, H.L. Clark, and Y.W. Lui, Phys. Rev. Lett. **82**, 691 (1999).
 - [54] O. Bohigas, A.M. Lane, and J. Martorell, Phys. Rep. **51**, 267 (1979).
 - [55] G. Colò, N. Van Giai, J. Meyer, K. Bennaceur, and P. Bonche, Phys. Rev. C **70**, 024307 (2004).
 - [56] B.K. Agrawal and S. Shlomo, Phys. Rev. C **70**, 014308 (2004).
 - [57] T. Sil, S. Shlomo, B.K. Agrawal, and P.-G. Reinhard, Phys. Rev. C **73**, 034316 (2006).
 - [58] C. Xu, B.A. Li, and L.W. Chen, Phys. Rev. C **82**, 054607 (2010).
 - [59] P.R. Bevington and D.K. Robinson, *Data reduction and error analysis for physical sciences* (3ed.) (McGraw-Hill, New York, 2003), p. 71.
 - [60] I. Tews *et al.*, Phys. Rev. Lett. **110**, 032504 (2013).
 - [61] A.W. Steiner and S. Gandolfi, Phys. Rev. Lett. **108**, 081102 (2012).
 - [62] C.M. Tarbert *et al.*, Phys. Rev. Lett. **112**, 242502 (2014).
 - [63] S. Abrahamyan *et al.*, Phys. Rev. Lett. **108**, 112502 (2012).
 - [64] J. Xu, L.W. Chen, B.A. Li, and H.R. Ma, Phys. Rev. C **79**, 035802 (2009); Astrophys. J. **697**, 1549 (2009).